Failure Behavior of Precracked Concrete Beams Retrofitted with FRP

By Oral Buyukozturk,[1] Member, ASCE, and Brian Hearing,[2] Student Member, ASCE

Abstract: The addition of fiber-reinforced plastic (FRP) laminates bonded to the tension face of concrete girders is becoming an attractive solution to the rehabilitation and retrofit of damaged structural systems. Flexural strength is enhanced with this method but the failure behavior of the system can become more brittle, often involving delamination of the composite and shear failure of the girder. This study first reviews failure modes including delamination with the use of FRP to rehabilitate various concrete structures and discusses methodologies used to characterize the failure processes of the system. Then, failure modes observed in an experimental program involving FRP-laminated laboratory models are reported. Physical models of reinforced concrete beams with variations in shear strengths, bonded laminate lengths, and epoxy types are precracked, then retrofitted with glass and carbon fiber-reinforced plastics and tested in an experimental program. Strengths are shown to increase with the addition of FRPs, but the specimens are observed to fail through a variety of mechanisms. Parameters affecting these failure modes are discussed, and techniques used in the analysis of these modes are reviewed. Results of this review are used to make recommendations for future work in the understanding of FRP retrofitted concrete behavior.

INTRODUCTION

The rapid deterioration of infrastructure is becoming a principal challenge facing concrete and bridge industries worldwide. Traditional structural rehabilitation methods such as external posttensioning and bonded steel plates often suffer from inherent disadvantages ranging from difficult application procedures to lack of durability, leaving the growing repair and rehabilitation market in need of cost-effective and efficient restoration techniques. Advances in the fields of plastics and composites have resulted in the development of high-strength, fiber-reinforced plastics (FRP) that offer great potential for lightweight, cost-effective retrofitting of concrete structures, including bridges. These high-performance materials can be bonded to the tension face of concrete members to increase the strength and stiffness of the structure with savings in application costs and improved durability over conventional methods. Recently, the use of FRP bonded to deteriorated, deficient, and damaged reinforced concrete structures has gained popularity in Europe, Japan, and North America.

The application of FRP as external reinforcement to concrete infrastructure has been studied by many groups. FRPs have been used to retrofit concrete members such as columns, slabs, beams, and girders in structures such as bridges, parking decks, smoke stacks, and buildings. Among these, the application of FRP to strengthen concrete beams has perhaps received the most attention from the research community; many researchers have reported improvements in strength and stiffnesses of retrofitted beams. Theoretical gains in flexural strength using this method can be significant; however, researchers have also observed new types of failures that can limit these gains. These failures are often brittle, involving delamination of the FRP, debonding of concrete layers, and shear collapse, and can occur at loads significantly lower than the theoretical strength of the retrofit system. Thus, there is a need for an improved understanding of these and other failure mechanisms of laminated concrete beams.

[1]Prof., Dept. of Civ. and Envir. Engrg., Massachusetts Inst. of Technology, Cambridge, MA 02139-4307.

[2]Res. Asst., Dept. of Civ. and Envir. Engrg., Massachusetts Inst. of Technology, Cambridge, MA.

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Researchers have indicated that failure criteria for laminated systems need to be established (Saadatmanesh and Ehsani 1989; Triantafillou and Deskovic 1991). Effective application of FRP to concrete is not possible until a fundamental understanding of the mechanics and failure mechanisms of the retrofit system is available. This paper first reviews failure mechanisms and current analytical techniques in the retrofit of concrete structures through studies on concrete beams externally reinforced with FRP sheets. Then, failure modes observed in an experimental program involving FRP-laminated precracked concrete beams are reported. Finally, techniques used in the analysis of these systems are summarized, along with current topics of research.

FRP IN RETROFITTING REINFORCED CONCRETE

FRPs exploit the advantages of high tensile strength fibers and are characterized by excellent corrosion resistance, fatigue resistance, low densities, and high specific stiffness and strength (Meier 1992). Commonly used fibers include E-glass, Kevlar/aramid, and carbon; these can be preimpregnated in matrices, lined unidirectionally in tow sheets, or woven into bidirectional fabrics. Application usually involves preparation of the concrete surface through mechanical or chemical means followed by a primer. If a fabric or tow sheet is used, an epoxy is applied to the concrete followed by the fiber in a process called "wet layup." Here, the adhesive is also the matrix, creating a stronger bond but also subjecting the fibers to debonding stresses with uneven concrete surfaces (Kaiser 1989). Preimpregnated FRP sheets and plates may be roughened, then attached to the concrete with an epoxy layer. Here the choice of adherent stiffness is crucial for effective stress transfer to the laminate. Thicknesses of installed plies usually range from 1 to 3 mm.

The implementation of this technique in strengthening existing reinforced concrete infrastructure has been demonstrated around the world. First applications of FRP to concrete structures include the use of carbon fiber-reinforced plastic (CFRP) laminates to strengthen a bridge with a damaged prestressing tendon; a mobile platform was used at night to restore the integrity of the bridge (EMPA 1994). Glass fiber-reinforced plastic (GFRP) sheets were used to retrofit a roadway bridge to reduce the steel stresses in the tendon couplers (Rostasy et al. 1992). GFRP tow sheets have been used on a bridge to increase the bending moment capacity (Nanni 1995); tow sheets have also been used on a prestressed box beam bridge (Finch et al. 1994).

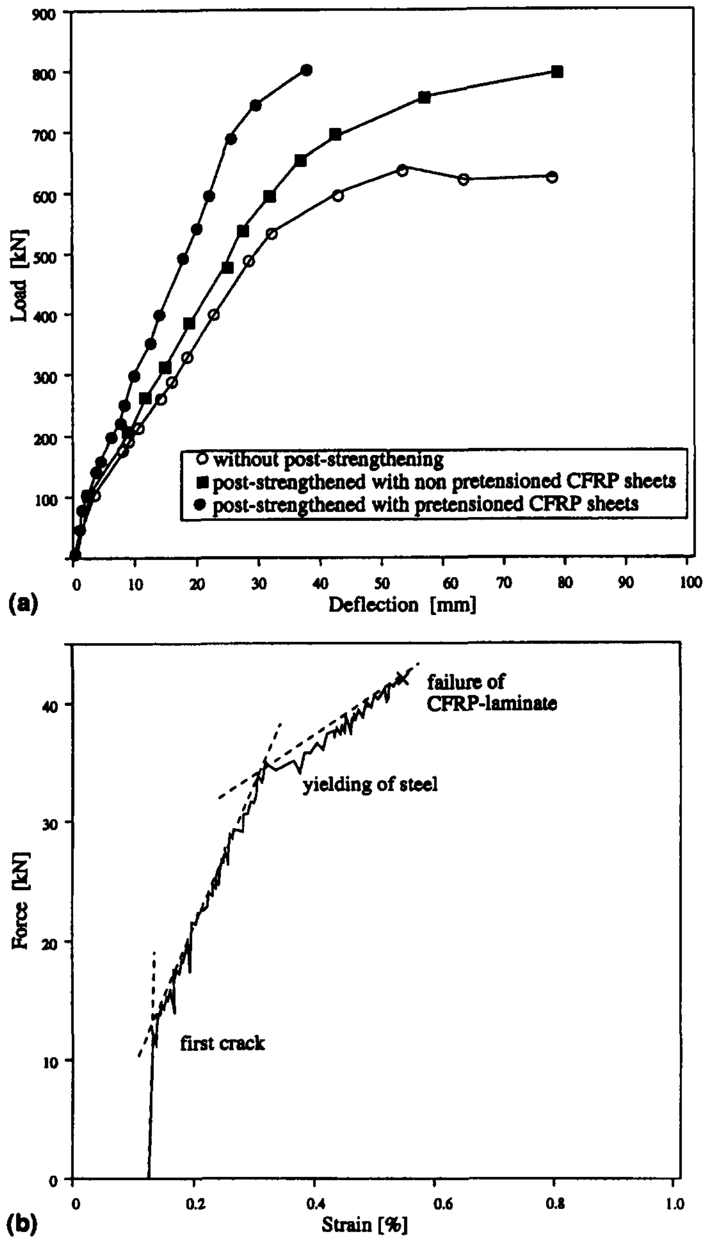


FIG. 1. Performance of Retrofitted Concrete Beams: (a) Increases in Stiffness and Strength (Kaiser 1989); (b) Force versus Strain in Nonpretensioned CFRP Composite Applied to Concrete Beams (Meier and Kaiser 1991)

Considerable research has been conducted to study the implications of retrofitting reinforced concrete beams with FRP. Early research has demonstrated that the addition of CFRP laminate to reinforced concrete T-beams can increase ultimate strengths by 22% and also increase stiffnesses of the beams; additionally, prestressed CFRP laminates can be used to further increase the stiffness of the system [Fig. 1(a)] (Kaiser 1989). The strain in the CFRP in different beams was shown to exhibit three distinct stages of beam behavior under loading, corresponding to the uncracked section of the beam, the cracked section with elastic steel, and finally the section with plastic steel, ending when the FRP failed in tension [Fig. 1(b)] (Meier and Kaiser 1991). Increases in strengths of over 40% and increases in stiffnesses were also reported with the use of glass and Kevlar-based FRP (Rostasy et al. 1992; Chajes et al. 1994, 1995a), where shear failures of the concrete beams were observed in addition to laminate tension rupture. In some cases, strength increases of up to 245% have been achieved through the use of external clamps to prevent debonding of the FRP (Saadatmanesh and Ehsani 1990). Additionally, prestressing the FRP laminate before application has been investigated (Triantafillou et al. 1992); it was found that high levels of prestress can result in shear failure in the concrete at the anchorage zone. These early studies, among others, have investigated a variety of retrofit systems and presented their responses under loading; many researchers have concluded that failure processes and their governing criteria need further study.

FAILURE MODES OF FRP RETROFITTED REINFORCED CONCRETE

Researchers have concluded that failure criteria for laminated systems need to be established (Ziraba et al. 1994). For this, however, a thorough understanding of the behavior of these systems is necessary. Many studies have presented a wide variety of failure modes observed in retrofit concrete beams (Meier 1995; Chajes et al. 1994); these failure types can be grouped into six distinct categories (Fig. 2). The criteria for each of these failures are affected by various parameters in the design of a FRP retrofit concrete beam. For example, it has been recommended that failure of these systems should occur with yielding of steel and ultimately rupture of the laminate before compressive concrete failures [Fig. 2(a)] (Meier 1995). This can be accomplished by optimizing the FRP and the steel-reinforcement ratio through traditional reinforced concrete design methods. Some other modes of failure, such as shear and debonding failures, can depend on other parameters such as existing shear reinforcement, crack configuration prior to strengthening, laminate length, and relative laminate/adherent/concrete stiffnesses.

Recently, exploratory studies investigating the influence of these and other parameters on the failure behavior of laminated reinforced concrete beams were performed (Vichit-Vadakan 1997; Morin 1997). FRP retrofitted precracked reinforced concrete laboratory specimens (Fig. 3), were tested in an experi-

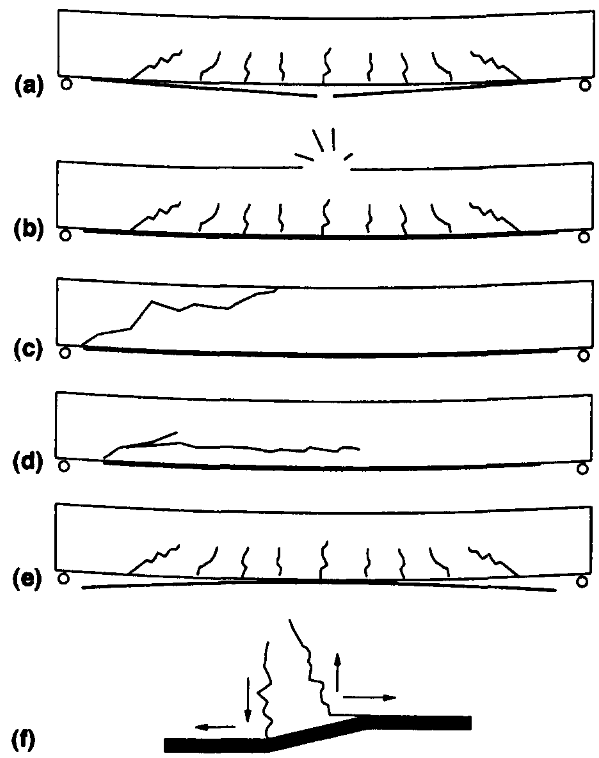


FIG. 2. Failure Modes in FRP Retrofitted Concrete Beams: (a) Steel Yield and FRP Rupture; (b) Concrete Compression Failure; (c) Shear Failure; (d) Debond of Layer along Rebar; (e) Delamination of FRP Plate; (f) Peeling due to Shear Crack

mental program. The beam specimens were first precracked at a maximum deflection of $l/240$, then retrofitted with two types of laminates, a 33% glass/nylon plastic (Grade 1600) and a 70% carbon/graphite (T700) plastic, respectively. Shear reinforcement spacings were varied at 13 and 18 cm to test different shear capacities of the beam. High-modulus and low-modulus epoxies were tested with two lengths of laminate, 90 and 70 cm, to investigate effects of retrofitting configurations. The thickness of the applied epoxies ranged from 0.5 to 2 mm, due to the varying roughness of the beams. These testing variables caused the specimens to fail in a variety of modes; observed modes of failure include flexural compression of the concrete, shear failure in the concrete beam, delamination of the FRP, and debonding of a layer of concrete at the flexural steel level. Sample load-midspan deflection curves for these failures are shown in Fig. 4.

Flexural failures of the retrofitted section include tensile rupture of the laminate [Fig. 5(a)], and compression failure of the concrete [Fig. 5(b)]. Both failures occur in a brittle manner with a sharp, explosive fracture. Strengths were increased by an average 23% in this program, though theoretical increases of up to 245% have been reported (Saadatmanesh and Ehsani 1990; Meier 1992). Deflections in the beam were reduced compared to the unretrofitted section, and the beam stiffness was increased. The ultimate strengths and stiffnesses of the beams with these flexural failures can be accurately predicted using strain- or stress-compatibility theories (introduced in the next section). However, it was found that flexural failures occurred only in beams with the long (90 cm) laminate length and higher shear reinforcement (13-cm spacing).

Shear failures of retrofit sections occur when the shear capacity of the section is exceeded prior to the load level reaching the flexural strength (Fig. 6). It is concluded that the FRP along the bottom of the beam does not significantly add to the shear strength of the section. These failures were achieved with the wider shear stirrup spacing (18 cm), which gave rise to a lower shear capacity. Shear cracks typically extended from the end of the laminate (70 and 90 cm lengths) to the point of loading. Shear failure occurred at relatively low applied loads, but these loads agreed with traditional reinforced concrete shear-strength theory. Research into improving the shear capacity of beams through innovative retrofit with FRP is increasing (Berset 1992; Sharif et al. 1994; Chajes et al. 1995b).

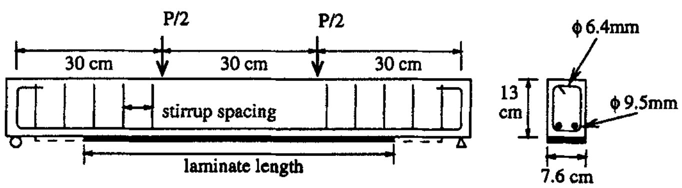


FIG. 3. Specimen Used in Testing Program

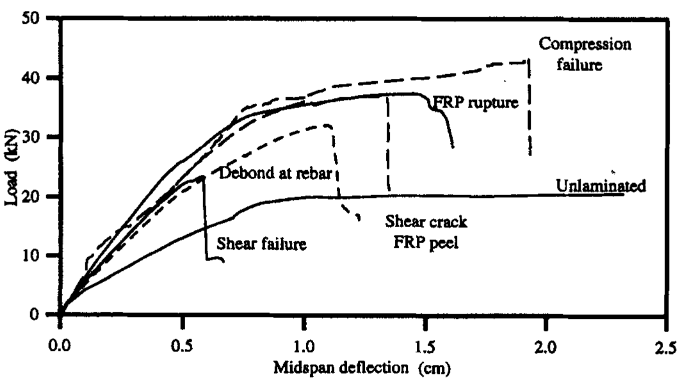


FIG. 4. Sample Load versus Deflection Curves for Specimens

Other types of failures in FRP retrofit concrete involve a variety of debonding mechanisms; these include failure of the concrete layer between the FRP and the steel [Fig. 7(a)], and delamination or "peeling" of the FRP from the concrete [Fig. 7(b)]. These failures typically occurred in beams with greater shear resistance (13-cm spacing) but were very brittle. Debonding of the concrete layer at the rebar occurred in beams with the shorter laminate length (70 cm), indicating that significant stress concentrations can occur at the laminate anchorage zone. Shear crack peeling of the laminate occurred with longer laminate lengths where significant shear cracks had formed. These debonding failures have been reported by a number of research teams (Ritchie et al. 1991; Saadatmanesh and Ehsani 1989; Sharif et al. 1994), and the laminate-concrete interfaces have been concluded to be susceptible to relative vertical displacements of shear cracks in the concrete beam (Meier 1992).

Thus, it has been shown that addition of FRP laminate can result in failure modes other than flexural failure and that these shifts in failure modes can alter the strength and ductility of the system. Prevention of brittle failure is an important criterion for safe and effective retrofit engineering; thus, all facets

(a)

(b)

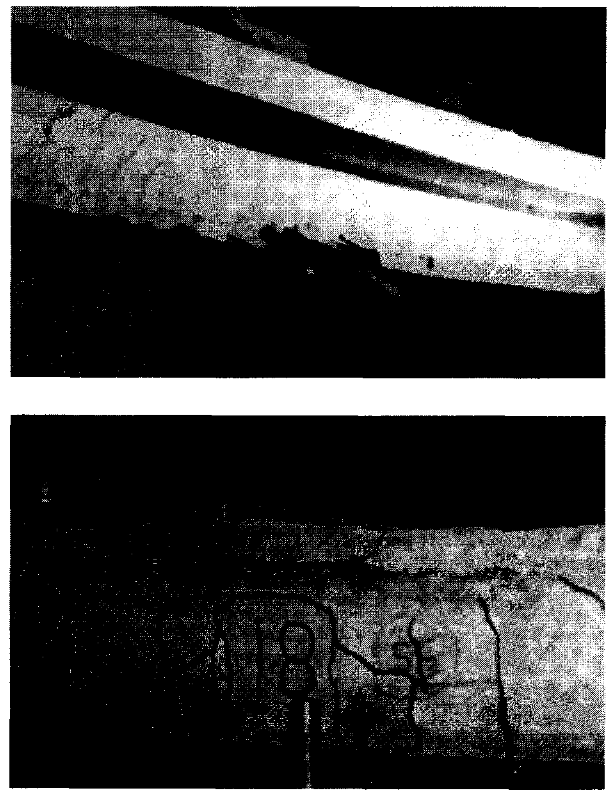
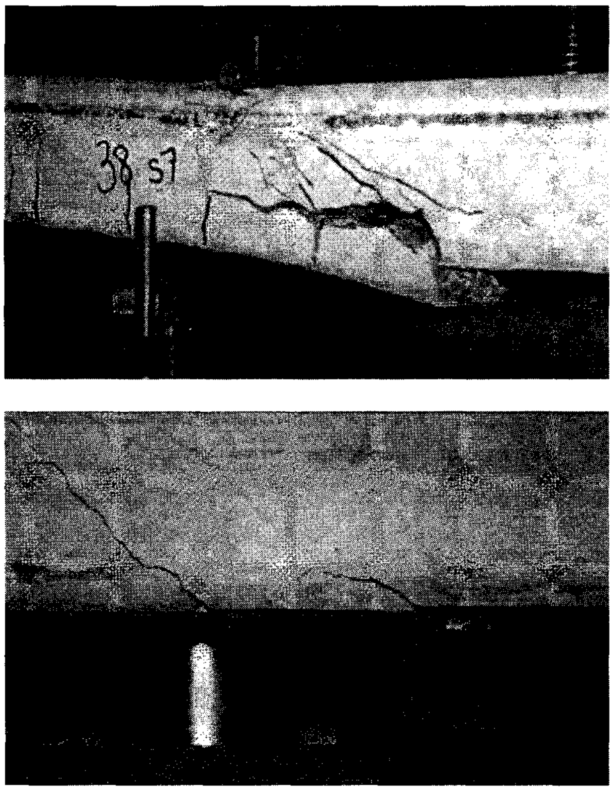


FIG. 5. Flexural Failures: (a) Laminate Tensile Failure (EMPA 1994); (b) Compression Failure

FIG. 6. Shear Failure



(a) (b)

FIG. 7. Debonding Mechanisms of Retrofit Beams: (a) Debond at Rebar Layer; (b) Shear Crack Peeling

of the performance of a retrofit system must be known, including the failure behavior. Therefore, analytical techniques for the evaluation of the behavior of the system through all possible failure modes must be made available to the retrofit engineer. In the next section, some existing techniques used in the analysis of these systems will be reviewed and areas requiring further development will be highlighted.

ANALYSIS OF FAILURE MODES

Considerable research has been conducted into the analysis of failure mechanisms of laminated reinforced concrete beams. These analyses usually consider short-term behavior of singly reinforced concrete members with rectangular cross sections strengthened with FRP on the bottom face of the member; in principle, similar approaches may apply for other geometric configurations.

Flexural Failures

Many researchers have analyzed the flexural behavior and strength of the composite cross section (Kaiser 1989; Triantafillou and Deskovic 1991; An et al. 1991; Karam 1992). Most studies have concluded that a stress or strain equilibrium of the section is applicable in the analysis of tensile failure of the laminate at an ultimate bending moment M_u of

$$\frac{M_u}{bd^2 f_c'} = \frac{f_y}{f_c'} \rho_s \left(1 - \frac{\bar{y}}{d}\right) + \frac{E_{fc}\varepsilon_{fc}^*}{f_c'} \rho_{fc} \left(\frac{h_p}{d} - \frac{\bar{y}}{d}\right) \quad (1)$$

where $\bar{y}$ = distance from the centroid of the concrete stress distribution to the top fiber; h_p = depth to the FRP; ε_{fc}^* and E_{fc} = ultimate strain and modulus of the fiber composite; and other terms are those used in conventional concrete analysis (ACI Committee-318 1995). Using similar arguments, these studies have also concluded that the section will fail in compression when the steel and/or FRP reinforcement ratio is large and the concrete strain exceeds 0.003. The ultimate moment M_u is then given by

$$\frac{M_u}{bd^2 f_c'} = 0.7225 \frac{c}{d} \left(\frac{h_p}{d} - 0.425 \frac{c}{d}\right) - 0.003 \frac{\left(1 - \frac{c}{d}\right)}{c/d} \frac{E_s}{f_c'} \rho_s \left(\frac{h_p}{d} - 1\right) \quad (2)$$

where c = neutral axis depth.

Shear Failures

It has been suggested that the shear capacity of FRP-strengthened concrete beams does not change significantly from those for the unretrofit beams (Plevris et al. 1995; Vichit-Vadakan 1997). Traditional analytical techniques have been presented based on conventional reinforced concrete shear theory (Ziraba et al. 1994)

$$V_{up} = \frac{1}{6} (\sqrt{f_c'} + 100\rho_s) bd + k \frac{A_v f_y d}{s} \quad (3)$$

where the coefficient k depends on variables associated with concrete cover rip-off (Roberts 1989). This analysis does not consider new shear-reinforcement retrofit techniques (Berset 1992; Sharif et al. 1994; Chajes et al. 1995b), where new methods in the analysis of these schemes must be developed.

Debonding Failures

The application of the laminate to the soffit of the concrete beam introduces shear transfer to the concrete/epoxy interface. At the termination of the laminate, a change in stiffness and discontinuity of beam curvature create a stress concentration in the concrete, often initiating cracks that can lead to debonding. Cracks initiating at the laminate anchorage area often follow two paths [Figs. 2(d) and 2(e)]. These failure mechanisms are often labeled as a shear-tension failure in the concrete leading to debonding along rebar or peeling due to shear crack mouth displacements leading to delamination along the concrete-laminate interfaces.

Shear-tension failure in the concrete initiating at the FRP sheet end can often lead to beam shear failure or debonding of a concrete layer at the flexural rebar. These complex failure mechanisms have been studied (Kaiser 1989; Arduini et al. 1994; Vichit-Vadakan 1997), but effective simplified analytical techniques are not widely available. The shear transfer between the concrete and the FRP sheet [Fig. 8(a)], can be expressed through a second-order differential equation (Vichit-Vadakan 1997)

$$E_{fc} \frac{\delta^2 u_l}{\delta x^2} = -\frac{G_a(u_b - u_l)}{t_l t_a} \quad (4)$$

where E_{fc} and G_a = Young's and shear modulus of the laminate and epoxy, respectively; t_l and t_a = thicknesses; and u_b and u_l = displacements of the beam and the laminate, respectively. A moment curvature relationship governing displacements of the beam can be established

$$u_b = \int_0^x \frac{My}{E_c I} dx \quad (5)$$

where E_c = Young's modulus of the concrete; and I = equivalent moment of inertia. Eqs. (4) and (5) can be used in an iterative procedure to calculate the shear stress transferred between the beam face and the laminate. Fig. 8(b) shows predicted shear-stress concentrations at the ends of the laminate for varying lengths using this method. These techniques are useful to study how the stress intensity and crack initiation at the end of the laminate can influence the beam failure behavior.

Peeling of the FRP from the concrete can initiate from the ends of the laminate or from existing cracks in the concrete. Flexural cracks, located in regions of the beam with large moment, can initiate interfacial fracture in strong shear (Mode II) propagation between the concrete and FRP interface [Fig. 9(a)]. Crack mouths located in regions of the beam with mixed shear and moments can subject an interfacial crack to mixed mode loading [Fig. 9(b)]. Studies of bond strength (Chajes et al. 1996) and interfacial fracture resistance (Karbhari et al. 1997) have been initiated using a variety of techniques such as single-lap shear and peeling specimens (Fig. 10). Measurements of peeling fracture energy G from elastic work done by a peeling test versus a phase angle ψ, measuring relative shear versus tensile opening modes have been initiated (Karbhari et al. 1997).

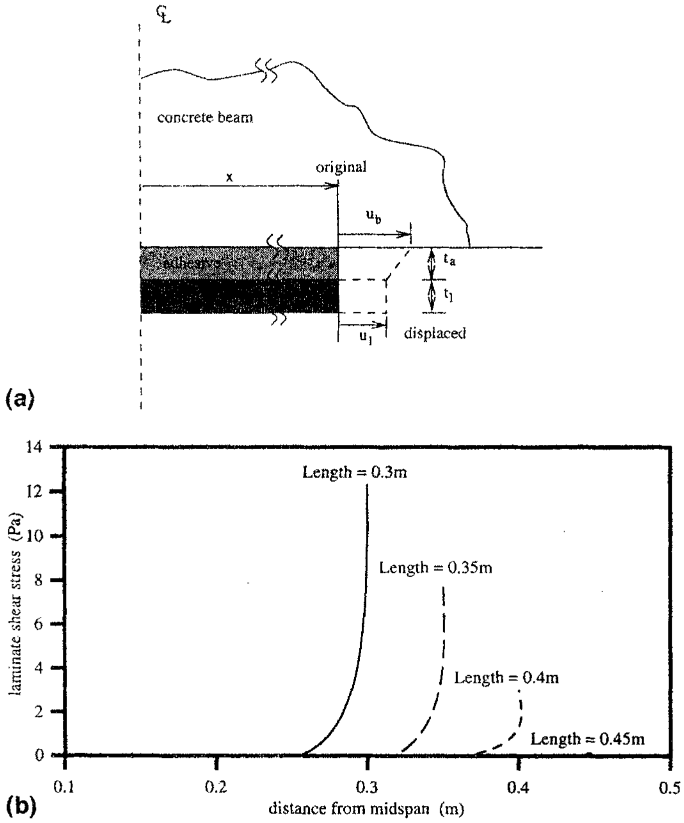


FIG. 8. Shear-Stress Transfer Distribution: (a) Shear Transfer between Concrete and FRP Sheet; (b) Shear-Stress Concentrations

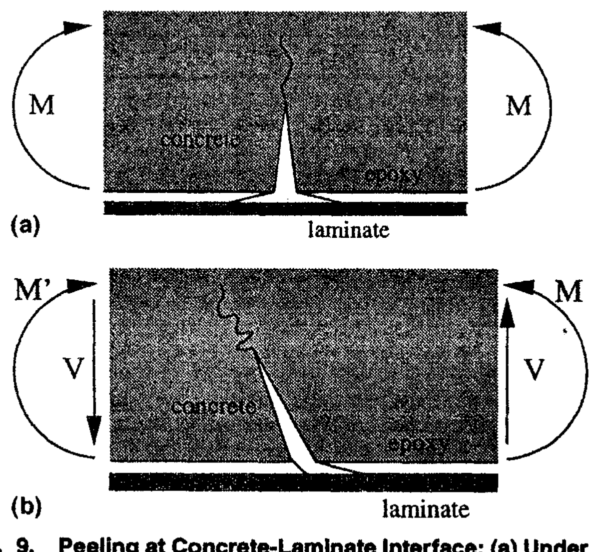


FIG. 9. Peeling at Concrete-Laminate Interface: (a) Under Moment; (b) Under Moment and Shear

Such an analysis can incorporate bimaterial constants in a modified fracture energy expression using interfacial fracture mechanics (Lee et al. 1995)

$$G = \frac{\left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)}{2\cosh^2\pi\varepsilon}|K|^2 \quad (6)$$

where K = mixed mode stress intensity factor

$$K = K_1 + iK_2 \quad (7)$$

and $\cosh^2\pi\varepsilon = 1/(1 - \beta^2)$. The complex interface stress intensity factor K has real and imaginary parts, K_1 and K_2, which

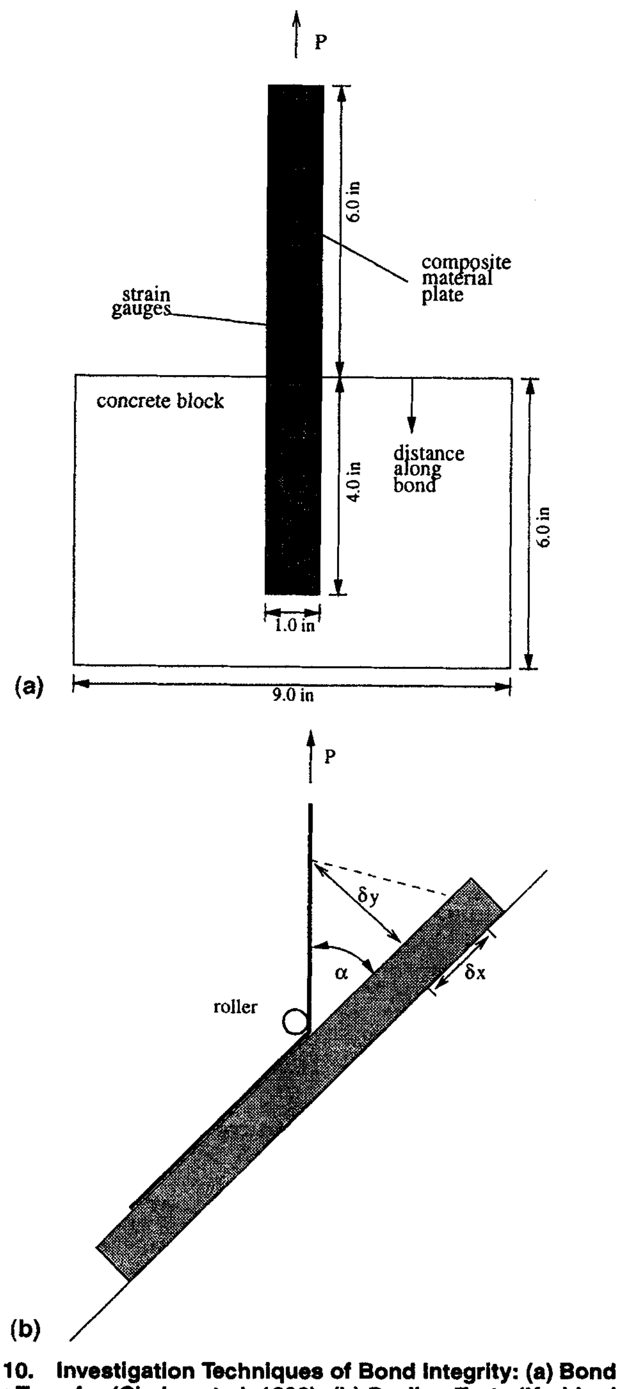


FIG. 10. Investigation Techniques of Bond Integrity: (a) Bond Shear Transfer (Chajes et al. 1996); (b) Peeling Tests (Karbhari et al. 1997)

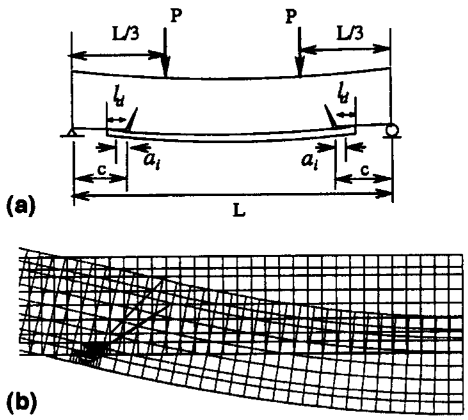


FIG. 11. Models Used to Investigate Peeling Mechanisms: (a) Initially Cracked Laminated Beam; (b) Refined Finite-Element Model

are similar to conventional Mode I and Mode II intensity factors in a homogeneous solid. Bimaterial elasticity depends on two moduli mismatch parameters α and β, defined by (Dundurs 1969)

$$\alpha = \frac{\bar{E}_1 - \bar{E}_2}{\bar{E}_1 + \bar{E}_2} \tag{8a}$$

$$\beta = \frac{1}{2}\,\frac{\mu_1(1 - 2\nu_2) - \mu_2(1 - 2\nu_1)}{\mu_1(1 - \nu_2) + \mu_2(1 - \nu_1)} \tag{8b}$$

where $\overline{E}_i = E_i/(1 - \nu_i^2) = 2\mu_i/(1 - \nu_i)$ for isotropic materials and related terms for anisotropic materials; and E_i, μ_i, and ν_i = Young's modulus, shear modulus, and Poisson's ratio, respectively for material *i*. The energy release rate *G* is defined as a function of the real phase angle $\hat{\psi}$ of the stress intensity factors ahead of the crack tip

$$\hat{\psi} = \arctan\left[\frac{\Im(KL^{i\varepsilon})}{\Re(KL^{i\varepsilon})}\right] \tag{9}$$

where *L* is a somewhat arbitrary reference length; and $\varepsilon = (1/2\pi)\ln(1 - \beta)/(1 + \beta)$. Here $\hat{\psi}$ measures the relative proportion of the effect of Mode II to Mode I stresses on the interface. For many practical systems β is small and can be neglected; the bimaterial phase angle reduces to

$$\psi = \arctan\left(\frac{K_2}{K_1}\right) \tag{10}$$

These concepts can be applied to study the delamination mechanism resulting from existing cracks in the concrete with the initially cracked laminated beams [Fig. 11(a)]. The complex stress intensity factor *K* can be investigated using advanced interfacial fracture mechanics combined with finite-element studies [Fig. 11(b)], and used to relate mesolevel fracture processes to global system behavior (Buyukozturk and Lee 1993; Lee et al. 1993). This type of approach represents an initial step in the evaluation of system performance incorporating real-life factors such as existing cracks and condition of the concrete-laminate bond.

CONCLUSIONS

The repair and rehabilitation of aging and deteriorating concrete bridges and infrastructure poses an urgent challenge for the civil engineering community. FRPs can play key roles in meeting these challenges. Advancements through research in this area are identified along with areas in need of further study. It is shown that a complete understanding of the effects these materials have on the performance of retrofit systems has not been achieved. In particular, various failure mechanisms that the retrofitted concrete beam can manifest, including the debonding and delamination mechanisms of FRP from the concrete beam, are not well known. More studies are needed to develop a better understanding of the shear capacity of retrofit sections, the effects in the anchorage regions of the FRP laminate, and failure mechanisms of debonding and delamination. Topics of future study should also include effects of material compatibilities and their resistances to degradation through both environmental and load cycles, and the assessment of retrofitted system integrity through the use of nondestructive evaluation. Quality assurance through nondestructive evaluation techniques (Buyukozturk 1997), is a rapidly growing field. These methods have the potential to provide quantitative verification of elements essential to effective rehabilitation such as adherent thicknesses, crack and delamination identification, and void detection.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A_v = shear reinforcement area;
b = beam width;
c = neutral axis depth;
d = depth to flexural steel;
E_c = modulus of concrete;
E_{fc} = modulus of fiber-reinforced composite;
E_s = modulus of steel;
f_c' = compressive strength of concrete;
f_y = yield strength of steel;
G = fracture energy;
G_a = shear modulus of adhesive;
h_p = depth to fiber-reinforced plastic;
K = complex interface stress-intensity factor;
M_u = ultimate bending moment;
s = shear reinforcement spacing;
$\Re$ = real part;
$\Im$ = imaginary part;
t_a = thickness of adherent;
t_l = thickness of fiber-reinforced laminate;
u_b = displacement of beam;
u_l = displacement of fiber-reinforced laminate;
V_{up} = shear strength;
$\bar{y}$ = depth to concrete stress centroid;
α, β = Dundurs' elastic moduli mismatch parameters;
ε = bimaterial oscillation index;
ε_{fc}^* = ultimate strain of fiber-reinforced composite;
μ_i = shear modulus of material i;
ν_i = Poisson's ratio of material i;
ρ_{fc} = fiber composite reinforcement ratio;
ρ_s = steel reinforcement ratio; and
$\hat{\psi}$ = complex stress-intensity phase angle.